Static Buckling of FML Columns in Elastic–Plastic Range

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The paper deals with the elasto–plastic buckling of thin–walled Fiber Metal Laminates short columns/profiles subjected to axial uniform compression. Structures of open and hollow (closed) cross-sections are considered built of flat plate walls. Multilayered FML walls are considered as built of alternating layers of aluminum and fiber–glass composite. Three elastic–plastic theories are employed for constitutive relations description of aluminum layers i.e. fully elastic material behavior, the J2–deformation theory of plasticity and the J2–flow theory later both with Ramberg–Osgood formula application, whereas composite layers are assumed elastic within whole loading range. Some exemplary results determined with the application of own analytical–numerical method based on the Koiter’s theory, in the Byskov and Hutchinson formulation are enclosed in the form of tables and plots.

Keywords: Fiber Metal Laminates, elasto–plastic buckling, thin–walled structures.

1. Introduction

The aeronautical engineering still has been looking for structural materials which fulfil tough requirements in terms of weight, strength, serviceable properties and economy. Last several decades have brought some material solutions which have attained these somewhat inconsistent demands. High strength alloys, as well as fiber-reinforced composites are widely applied in aircraft design despite some disadvantages as poor impact resistance and fatigue properties. Thus the fiber metal laminates are regarded as better materials combining the best features of fiber-reinforced composites and metals. At the first place comes their higher damage tolerance with superior fatigue behavior. These features and permanently weight saving lead however to interest in a stability analysis which still has been within the research topics. Possible sever conditions of aircraft members made of FML type
materials are also the reason of elastic-plastic deformations in the metallic layers and the source of buckling research in the elastic-plastic range.

In our authors’ team the elasto-plastic buckling of thin-walled structures has been the scientific topic of interest of late Professor Katarzyna Kowal-Michalska [21]. Also this aspect of research Professor Kowal-Michalska has initiated in our investigations of FML panels. Thus this paper is the result of our cooperation within common scientific activity devoted to elasto-plastic buckling of FML columns.

Fiber Metal Laminates (FMLs) are hybrid materials, built from alternately bonded thin layers of metal alloy and fiber reinforced epoxy resin. These materials are manufactured in autoclaving process by bonding composite plies to metal ones. FMLs, with respect to metal layers, can be divided into FMLs based on aluminum alloys (ARALL reinforced with aramid fibers, GLARE - glass fibers, CARALL – carbon fibers) and others. Nowadays material such as GLARE (glass fiber/aluminum) due to their very good fatigue and strength characteristics combined with the low density find increasing use in aircraft industry [19].

Within the recent literature one can find numerous publications concerning singular isolated plates of different isotropic material properties which work in the post-buckling elastic and elastic-plastic range but there are relatively few works dedicated to complex plate structures made of composite and/or laminate materials [2-4, 7, 16]. In the last years, due to widespread of professional Finite Element Method software application, several publications appeared where full force-shortening curves (until fracture) of structures were determined. It concerns structures with a complex cross section made of different materials - also including orthotropic material [9, 14, 15].

In few works [6, 7, 13, 17] one can find the solution to the stability problem of thin-walled columns made of isotropic and orthotropic materials in elastic-plastic range. In the current study analogous issue for multi-layered materials of Fiber Metal Laminate type is considered.

GLARE type FML consists of alternate thin aluminum sheets and unidirectional high-strength glass fiber layers pre-impregnated with adhesive. Usually each glass composite layer is composed of a certain number of unidirectional (UD) plies which are stacked either unidirectional, in a cross-ply or angle-ply arrangement [18].

![Figure 1: Open cross-sections analyzed columns](image-url)
number of layers, plies orientation and the stacking sequence of the UD plies in the entire FML panel depend on the GLARE grade. For example, a GLARE 2 has two UD plies in a particular composite layer with the same 0-degree orientation, while a GLARE 3 has two mutually perpendicular UD plies (cross-ply arrangement). The most common type of aluminum applied in GLARE is 2024-T3 Alloy [19].

When the plate structures made of GLARE is subjected to in-plane uniform compression in the elastic–plastic range of stresses, the buckling occurs in such a way that aluminum layers become plastic but the glass layers remain elastic. Therefore the behavior of such structures differs significantly from the behavior of pure aluminum ones.

2. Subject of Consideration

Under consideration were prismatic thin–walled structures built of FML plates connected along longitudinal edges creating profile/column member of hollow or open section presented in Fig. 1 and Fig. 2, respectively. The entire structure is subjected to uniform axial loading and both its loaded ends are simply supported. In order to account for all possible modes of global, local and coupled buckling, a plate model of thin–walled structure has been adopted. It was also assumed that both component materials the structure is made of obey the Hooke’s law.

3. Method of Solution

The problem of buckling in the elastic–plastic range of thin–walled FML profiles/columns, axially loaded by uniformly distributed compression, is investigated basing on the analytical–numerical method (ANM) elaborated for the analysis of the elastic stability of multi-layered thin–walled columns [8]. The nonlinear constitutive relations between stress and strain for a singular elastic-plastic component layer is derived on the basis of the J2–deformation theory of plasticity (i.e. DT) or the J2–flow theory (i.e. incremental theory of plasticity – IT) with Ramberg–Osgood formula application. Thus, these relations for material of FML metallic layers in the elastic range is simply defined as

$$\sigma = E\varepsilon \text{ for } \sigma \leq \sigma_0$$  \hspace{1cm} (1)
whereas the elastic–plastic stress-strain behaviour of FML aluminium layer is described by a Ramberg-Osgood representation of the following type [6, 7]

$$\sigma = \frac{(E - \sigma_Y)\varepsilon}{1 + \left(\frac{(E - \sigma_Y)\varepsilon}{\sigma_Y}\right)^n} + E_Y \varepsilon \quad \text{for} \quad \sigma \geq \sigma_0$$  \hspace{1cm} (2)

where: $\sigma$ – stress, $\varepsilon$ – strain, $E$ – Young’s modulus, $\sigma_0$ – proportional limit, $\sigma_Y$ – conventional yield limit, $E_Y$ – tangent modulus corresponding to the yield limit $\sigma_Y$, $n$ – exponent in the Ramberg-Osgood formula. The orthotropic composite layers are assumed to have only elastic properties due to the linear stress-strain characteristic up to fracture.

For any orthotropic plate the constitutive relationships for the elastic range and the elastic–plastic range have very similar or even identical form (Eq. 3) [7, 11]

$$\begin{align*}
ex &= K_{11}\varepsilon + K_{12}\varepsilon y, \\
\sigma_y &= K_{12}\varepsilon x + K_{22}\varepsilon y, \\
\tau_{xy} &= K_{33}\gamma_{xy}
\end{align*}$$  \hspace{1cm} (3)

Therefore comparing the corresponding coefficients in both relations, the instantaneous conventional parameters of ‘elastic composite’ for particular layers of entire FML structure can be found out. In a consequence the problem of inelastic stability of FML structures can be analyzed in the analogous way as the problem of elastic composite structures. The coefficients $A_{11}$, $A_{33}$ in (Eq. 3) determined on the basis of the $J_2$ – deformation (DT) or $J_2$ – flow (IT) theory of plasticity depend on the appropriate Young’s modulus, secant and tangent moduli for the considered material layer characteristics in the elastic–plastic range.

The examined problem is solved in a numerical way. The elastic problem is solved based on the asymptotic Koiter’s theory [5], in the Byskov and Hutchinson formulation [1]. The solution of the first order approximation enables one to determine the values of buckling global and local loads and the corresponding buckling modes. This analytical–numerical method [7, 8, 12] created to solve the elastic buckling problem is applied here to calculate critical load values and buckling modes for inelastic thin–walled FML columns and panels response. For particular column geometrical dimensions and material data constants of singular FML layer, for an assumed number of buckling half–waves, the elastic buckling stress for the considered FML structure is calculated. The most important advantage of this method is that it enables one to describe a complete range of a buckling behaviour of thin–walled structures from a global (i.e. flexural, flexural–torsional, lateral, distortional buckling and their combinations) to a local stability, including a mixed buckling modes [7, 8, 12].

Further, a zero value of the function $f = f(\sigma - \sigma_e)$ is searched to apply the method of secants, where $\sigma_e$ is the value of the critical stress of the ”elastic orthotropic” structure. A reasonable accuracy is assumed during the computations, that is $\sigma \approx \sigma_e$ when $(\sigma - \sigma_e) \cdot 100%/\sigma \leq 0.01\%$.

The proposed method allows to consider the transition of buckling mode together with the increase of loading as distinct from the usual assumption that the elastic–plastic buckling mode is analogous to the elastic one.
For a given geometrical parameters, material constants of each FML layer and for the assumed number of buckling half-waves the elastic buckling stress for the considered composite structure is then calculated.

4. Exemplary Results of Obtained Solution

As some examples of employed method of solution to the elastic–plastic problem of thin–walled FML hybrid composite structure few complex plate structures have been considered (Fig. 1 and Fig. 2). It was assumed that the loaded edges of considered structure are simply supported. In order to account for all modes of global, local and coupled buckling, a plate model of thin–walled structure has been employed. The overall dimensions of selected structures were chosen in such a way that the stability loss occurs in the elastic–plastic range for aluminum layers.

In current paper the detailed analysis was performed for the four chosen FML members which overall and cross–section parameters were as follows:

- a beam/column profile with a square cross–section (Fig. 1a) and \( L = 1300 \) mm, \( b = 130 \) mm,
- a beam/column profile with a trapezoidal cross–section (Fig. 1b) and \( L = 1300 \) mm, \( b_1 = 100 \) mm, \( b_2 = 140 \) mm, \( b_3 = 140 \) mm,
- a beam/column profile with a top–hat (Fig. 2a) and a lip channel cross section (Fig. 2b); \( L = 1300 \) mm, \( b_1 = 130 \) mm, \( b_2 = 65 \) mm, \( b_3 = 15 \) mm.

In all presented cases \( L \) indicates the column length. Constructions under investigation are built of alternate aluminum sheets and unidirectional high strength glass fiber layers so assumed stacking sequence corresponds to GLARE 3 grade with 2024-T3 sheets [11], [20]. The total number of layers in considered column walls equals 13. That leads to the total wall thickness of column/panel wall equal to \( t = 4.3 \) mm where the thickness of singular aluminum sheets equals 0.4 mm and of particular doubled cross–ply fiber layer is 0.25 mm. Applied in computations mechanical properties of both isotropic layers are presented in Tab. 1 [10, 20].

<table>
<thead>
<tr>
<th>Material data of GLARE 3-7/6-0.4</th>
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<th>Plastic properties</th>
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<td>( E ) (GPa)</td>
<td>( \nu ) [-]</td>
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<td>( \nu ) [-]</td>
<td>( \sigma_0 ) [MPa]</td>
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<td>Aluminium 2024-T3</td>
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<td>Prepreg</td>
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Some chosen results of the critical stress $\sigma_{cr}$ calculations for the considered thin–walled FML structures (Figs. 1-2) are presented in the following figures, respectively. Applied into the analysis three theories are distinguished in these figures as: elastic theory EL, J2-deformation theory DT and J2-incremental theory IT. For individual member FML’s cross-sections a stability loss can occur under symmetry (S) and anti-symmetry (A) conditions along symmetry axis of the cross-section. In enclosed plots the determined critical stress values are presented as a function of the number of half-waves $m$ formed in the longitudinal direction. The lowest values of $\sigma_{cr}$ are summarized in Tables 2-6. The buckling modes of analyzed FML structures are also presented in Figs. 5, 6, 8, 9, 11, 12, 14-16.

Figure 3 Plots of buckling stress $\sigma_{cr}$ versus number $m$ of half-waves $m$ for a hollow column of square cross-section.
4.1. **Hollow cross-sections**

4.1.1. **Square cross-section**

The plots in Fig. 3 present critical stress values $\sigma_{cr}$ for the square cross-section from Fig. 1a. Particular curve corresponds to the particular plasticity theory and gives the $\sigma_{cr}$ value as a function of half-waves number in longitudinal direction of the compressed column. In Tab. 2 the lowest values of critical stresses for considered symmetry conditions on column symmetry axis are shown for comparison. Critical stress values $\sigma_{cr}$ for symmetry conditions (S) are lower than for anti-symmetry conditions, as it was expected. The lowest value of critical stress $\sigma_{cr}$ was obtained when the deformation theory (DT) formulation was applied. From Tab. 2 it is clearly visible that the number of half-waves corresponding to the lowest value of $\sigma_{cr}$ is different for elastic theory (i.e. $m = 14$) from those of deformation theory (i.e. $m = 13$). Both local buckling modes determined for considered theories are very similar for assumed boundary conditions (Figs. 4, 5).

<table>
<thead>
<tr>
<th>Table 2 Square cross-section</th>
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<td>$\sigma_{cr}$ [MPa]</td>
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**Figure 4** Shapes of local antisymmetric (A) buckling modes for elastic (EL) and inelastic range (DT, IT) for square cross-section
4.1.2. Trapezoidal cross-section

For the trapezoidal cross-section from Fig. 1b, there are critical stress values $\sigma_{cr}$ as a function of half-waves number in longitudinal direction presented in Fig. 6 for all considered theories (with both plasticity ones). After that, in Table 3 the lowest values of critical stresses for considered boundary conditions on symmetry axis are given. The local buckling modes for assumed boundary conditions are shown in Fig. 7 and 8. The conclusions from the elastic–plastic analysis of FML columns of the trapezoidal cross-section are very similar to the comments formulated above for the square cross-section FML column. When the final results of square and trapezoidal cross-section columns are compared one can observed that the critical stress values are lower for a trapezoidal–cross section column.
Figure 7 Shapes of local antisymmetric (A) buckling modes for elastic and inelastic range for trapezoidal cross-section

<table>
<thead>
<tr>
<th>Table 3 Trapezoidal cross-section results</th>
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Figure 8 Shapes of local symmetric (S) buckling modes for elastic and inelastic range for trapezoidal cross-section
4.2. Open cross-sections

In the case of examined open cross-section columns/profiles (presented in Fig. 2) i.e. top hat and lipped channel, for assumed their overall dimensions all global buckling modes should be examined during the analysis. It means that flexural mode (S), distortional–flexural mode (S), flexural–torsional mode (A), distortion–flexural–distortional mode (A)) and local buckling mode including distortional–local modes, should be taken into account. Therefore additional indication is introduced for open cross-section profiles - global buckling mode (i.e. $m = 1$) is denoted by G and local buckling mode (i.e. $m \geq 1$) by L.

![Figure 9](image1)

**Figure 9** Buckling stresses $\sigma_{cr}$ versus number of axial half-waves $m$ for top hat

![Figure 10](image2)

**Figure 10** Shapes of global buckling modes for top hat
4.2.1. Top hat

For the top hat cross-section columns/profiles (see Fig. 2 for cross-section shape) results of critical stresses as a function of half-waves number \( m \) are presented in Fig. 9. The lowest values of global and local critical stresses \( \sigma_{cr} \) are shown also in Tab. 4. As it can be seen in this case a flexural-torsional global buckling mode (i.e. \( m = 1, A \)) took place in the elastic range because the following relationship is fulfilled \( \sigma_{cr} = 97 \text{MPa} < \sigma_0 = 170 \text{MPa} \). While a flexural buckling is observed in the elastic–plastic range (i.e. \( m = 1, S \)). Following this observation the flexural global buckling modes could be named as "pure bending" (Fig. 10) while antisymmetry mode for elastic range is a distortional–flexural–torsional mode because the lips are not perpendicular to the flanges (see EL-A_G curve in Fig. 10).

It can be seen in Table 4 that the value of the local critical stress \( \sigma_{cr} \) of symmetric mode (i.e. \( m = 4, S \)) for elastic range is lower in comparison to a local antisymmetric mode buckling stress (i.e. \( m = 2, A \)) for elastic range. However, values of \( \sigma_{cr} \) for both elastic–plastic formulations and antisymmetrical modes are lower than the symmetric ones. For \( m \geq 1 \) buckling modes are distortional–local modes for both boundary conditions (Fig. 11, Fig. 12). Buckling modes are practically the same for each of applied theories.
Figure 13 Buckling stresses $\sigma_{cr}$ versus number of axial half-waves $m$ for lip channel column

Figure 14 Shapes of global buckling modes for lip channel

Figure 15 Shapes of local anti-symmetric (A) buckling modes for lip channel
### Table 4 Top hat results

<table>
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<th>Conditions along symmetry axis of cross-section</th>
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<th>Elastic–plastic range</th>
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<th>$\sigma_{cr}$ [MPa]</th>
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**Figure 16** Shapes of local symmetric (S) buckling modes for lip channel

### Table 5 Lipped channel results

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4.2.2. Lipped channel

In Fig. 13 critical stress values $\sigma_{cr}$ as a function of half–waves number $m$ are presented for the FML column of lipped channel cross–section. Tab. 5 shows also the lowest values of global and local $\sigma_{cr}$ for both considered boundary conditions while corresponding to them buckling modes are given in Figs. 14–16.

The lowest value of critical stresses $\sigma_{cr} = 128$ MPa corresponds to a global flexural–torsional mode (i.e. $m = 1$, A) in elastic range (see EL_A_G line in Fig. 14). The global buckling stress value $\sigma_{cr} = 198$ MPa ($m = 1$, S) corresponds to a distortional–flexural buckling mode for elastic range (Fig. 14). Symmetric global buckling modes are similar for considered constitutive theories. Local buckling stress values are lower for symmetric modes in comparison to anti-symmetry ones.

Presented in Figs 15 and 16 buckling modes are of distortional-local anti-symmetric and symmetric type. It should be emphasized that local symmetric buckling modes (Fig. 16) differ slightly between themselves at the junction of flanges with the lips.

As it can be seen from presented in current work buckling modes for FML multi–layered structures determined buckling modes differ at least slightly between themselves because particular elastic glass fibre layers work within elastic range. Thus mechanical properties of glass fibre layer remain unchanged in elastic–plastic range of entire FML wall, when aluminium layer changes own properties from isotropic to orthotropic. It makes that multi–layered structures are not as sensitive to changes of buckling modes as one–layered structures. The latter change their mechanical properties across whole thickness in the elastic–plastic range [6, 13].

5. Conclusions

In work the comparison of critical stresses for thin–walled FML structures in elastic and elastic-plastic range is presented. Two plasticity theories were considered i.e. J2-deformation theory and J2-incremental theory. The lowest values of critical stresses for all analysed structures were obtained in elastic–plastic range for the deformation theory. It is fully consistent with results presented in literature survey. Moreover it ought to be pointed out that:

- the solutions given here are valid in the cases of the uniform compression of the thin–walled FML structure. Other types of loadings would need further investigation,
- the usual assumption, made in many works in the field, that the buckling modes in the elastic and elastic–plastic range are identical cannot be true in some cases,
- it should be noted that the buckling modes in elastic and elastic-plastic range can be not always cover–up.

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