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Design of Retraction Mechanism of Aircraft Landing Gear

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A computer aided modeling of a nose wheel landing gear mechanism of a light airplane is under consideration. The design process is carried out under the assumption that the exact volume in the fuselage for the gear is defined. The Solid Edge system is used in order to calculate the forces appearing during landing and retraction of the gear. The dynamics of the gear during retraction is conducted by using finite element technique with especially chosen shape function for describing rigid body motion of the mechanism. The mechanism is modeled as a planar four bar linkage with rigid links. The computer simulation of the gear motion by using the method presented in this paper is compared with the results obtained from analysis in ADAMS system. The model of friction in revolute and prismatic joints is assumed and its influence on system dynamics is discussed.

Keywords: Landing gear, finite element method, rigid links, four bar linkage, retraction mechanism

1. Introduction

Design of retraction mechanism of aircraft landing gear is a very responsible area. The geometry and kinematics of the gear are functions of the parameters of the aircraft and usually should be designed for every type of aircraft independently. In the design process it should be taken into account the individual parameters of the aircraft such as weight, space, volume assigned for the gear, as well as aircraft's mission, such as fighter, transport passenger or cargo [10]. The gear must be able to carry the impacts during landing since the devastation of a landing gear can cause serious accident of the aircraft [1, 11].

The design of landing gear begins from the study of conception of shock absorber and retraction system. The data needed in the analysis is: the mass of the airplane, volume for landing gear in the fuselage and the mounting points for connection between landing gear and fuselage frame structure. Next the loadings of the system are determined and the appropriate calculation of strength of elements are provided. Usually the finite element method is used for static calculation and dynamics effects are taken into account by appropriately increasing loadings acting on the system according to FAR-23 regulations. The simulation of motion during retracting and extending of the gear is also of great interest to the designers.

In the paper the design of retractable mechanism of light transportation airplane [5] is presented. The kinematics and dynamics of the designed gear mechanism is considered with taking into account influence of friction on the motion of the mechanism. The general methodology was to use a two dimensional four bar linkage model with rigid links for the computer analysis. The especially chosen shape function for modeling rigid body motion in the finite element formulation [7, 8] is used and the results of calculations are compared with those obtained for ADAMS software. In the mechanism analysis the use of a multibody code such as ADAMS allow the designer to create models as complex as desired [5, 6].

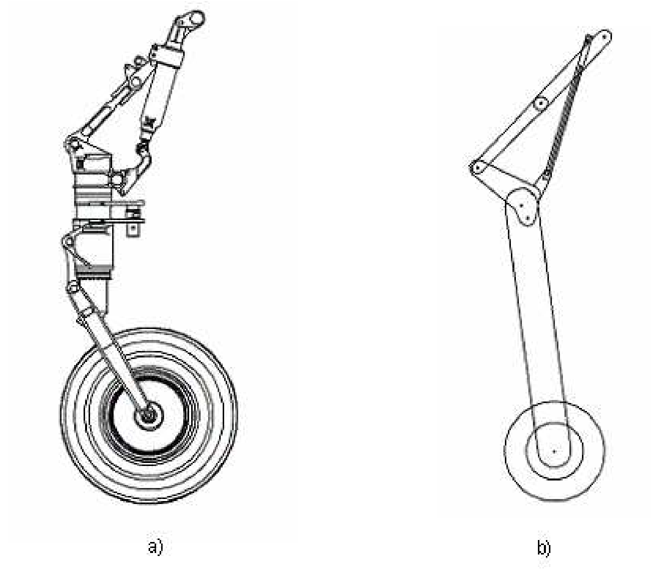

Figure 1 Constructional model of landing gear (a) and its beam–type model (b)

The schematic 2D drawing of the model of the retractable front gear is presented in Fig. 1a, and its stick diagram in Fig. 1b. Stick diagrams display only the essential skeleton of the mechanism which however embodies the key dimensions that determine the path of the motion [12]. In the presented paper the stick diagram is also used as the basis of a beam model built in order to compare the gear motion for two different methods used in the paper: finite element method applied for rigid links and the multibody method used in the ADAMS program. From Fig. 1 it can be clearly seen that the mechanism can be modeled as planar four bar linkage [3] with external torques applied to two links (Fig. 2): to the crank, and to the follower, since the actuator is connected by short arms with these two links.

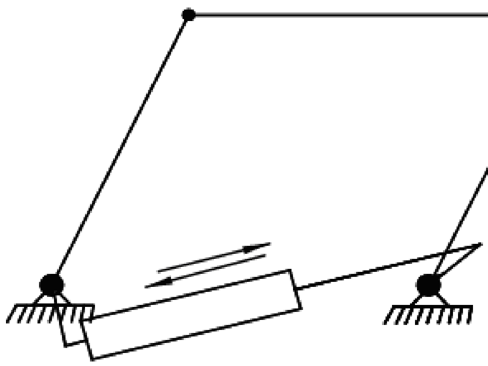

Figure 2 Four bar linkage modeling landing gear

2. Loadings acting on landing gear

The landing gear presented in the paper was designed for light transportation airplane of total mass equal to 8600 kg and the following initial data was assumed:

- the angle of rotation is equal to 120 deg,
- the mass of the gear is 64 kg,
- the direction of retraction: along axis of symmetry of the airplane, retraction in direction of the flight.

Design of the landing gear was conducted with the use of Solid Edge V7 program. The kinematic retraction system came into being on the basis of the four bar linkage (Fig. 2) with tracking element. Due to such solution it was possible to obtain the required revolution angle of the gear with small external overall dimension as well as small mass of the whole mechanism.

The force from the actuator is applied to two links i.e. to the crank and the follower – in the model presented in Fig. 2 the external torques are applied to nodes 1 and 4. The dynamic of the system is strongly influenced by the shock absorber and the wheel. The more precise model is presented in Fig. A1 (see Appendix) in which the shock absorber is taken into account by adding additional link l_5 permanently attached to the crank – the angle between this link and the crank is constant and is equal to $\xi = 2.4$ rad. The data for the model of the four bar linkage is presented in Tab. 1. The cross–sectional area of the beam model are assumed to be constant for each link; the approximate values were calculated with the use of model similarity principles. The additional data needed for calculation is the angle between assumed global coordinate system XY and gravity vector which is equal to $\pi/2 - \delta$, where δ = 1.13 rad.

Table 1 Four bar linkage parameters

Parameters	Crank	Coupler	Follower
Length, m	l_2= 0.1755	l_3= 0.217	l_3=0.34
Cross-section area, m^2	A_2= 1·10^{-2}	A_3= 4.256·10^{-4}	A_4= 1.452·10^{-3}
Ground: length, m l_1= 0.5729			
Shock–absorber: length, m l_5= 0.759			
Masses, kg: wheel: m_{5k} = 19.5; shock–absorber: m_5 = 36.8			
Modulus of elasticity, Nm^{-2} E = 2.1×10^{11}; Mass density, kgm^{-3} ρ = 7800			

The loading acting on the landing gear can be divided into the following categories: externally applied forces (torques), mass forces (gravity), aerodynamic forces and friction forces.

The external torques applied to the crank and the follower are computed based on the force in the actuator which is obtained from geometry and pressure parameters of the hydraulic steering system. In the numerical calculation this force (denoted by F in Fig. 2) was assumed as constant value of 30 000 N. The torques applied to links of the four bar linkage are function of the crank angle since during motion (i.e. during retraction and extending the landing gear) the arms of operation forces are changing. Next loading which should be taken into account is the overload of kinematic system - according to FAR-23 the gravity load is calculated for acceleration $3g$, where g is the gravitational acceleration.

During retraction and extending of the gear the air resistance force acts on the landing gear. This force can be determined in aerodynamic tunnel, or can be obtained from the formula:

$$P_x = \frac{1}{2}\rho v^2 S C_x \tag{1}$$

where: P_x- resistance force, ρ- mass density of the air – it depends on the altitude of flight and for the sea level it is equal to 1.225 kgm^{-3}, v- airplane velocity – assumed to be 70 msec^{-1}, S- cross section face area i.e. perpendicular to line of current. The surface S is decreasing with retraction of the landing gear. C_x stands for the coefficient of air resistance - it is often obtained experimentally and for the considered landing gear is assumed to be C_x = 0.5 [5].

The modeling of air resistance force can be effectively done by using the ADAMS program. Fig. 3 presents the change of air resistant force with the angle of gear rotation. Angle 0 degrees stands for the extended gear, 120 degrees represents the case of retracted gear.

Taking into account the above mentioned loadings it was possible to calculate the force needed in the actuator in function of the angle of gear rotation. For the designed model this force is presented in Fig. 4 for the G–load of 3g, and the graphs are presented for gravity forces and aerodynamic forces.

The motion of the landing gear during retraction is also of great interest. This problem was solved by using finite element technique applied for rigid links andindependentky by ADAMS system. The case is described in the next two chapters.

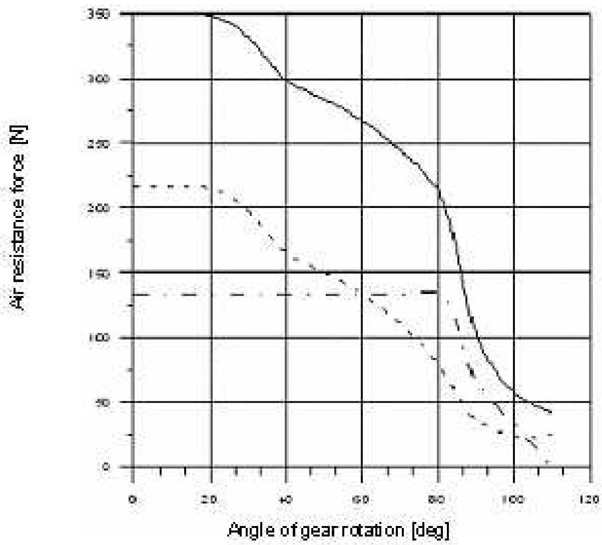

Figure 3 Course of air resistance with angle of gear rotation: (——) resistance of the whole gear, (- - - -) undercarriage leg, (– · – · –) wheel resistance

3. Analysis of landing gear motion by FEM

Traditionally the method of determining the rigid body motion of mechanisms is based on the classical analysis which involves using one of the method of determining equations of motion (e.g. Lagrange's, Gibbsa – Appel's or similar equations) and then applying it to the given example. Such method is very time–consuming since for each structure the equations of motion should be derived from the beginning. Moreover in case of external torques applied to more than one link of the mechanism (e.g. retraction kinematic system of aircraft landing gear) it is necessary to transform the external loadings to generalized coordinates. This procedure is sometimes more complicated than the derivation of the equations of motion itself. The equations of motion for the given example of the four bar linkage are presented in the Appendix.

The discrete methods such as finite element method are free of this disadvantage. The method used in the present paper is based on the finite element technique and obtained equations are solved by using Newmark method. The shape function for rigid body elements is taken from the earlier publications [7] and the equations of motion for structures with rigid links are derived. The method of obtaining rigid body motion is presented in the closed form and the system equations can be solved by using the same procedures as for finite element analysis.

Figure 5 presents a general planar rigid element in two frames of reference: the global coordinate system XY and the element oriented frame xy. The figure shows schematically two positions of an element: the initial position 12, and the position due to rigid body motion 1'2'.

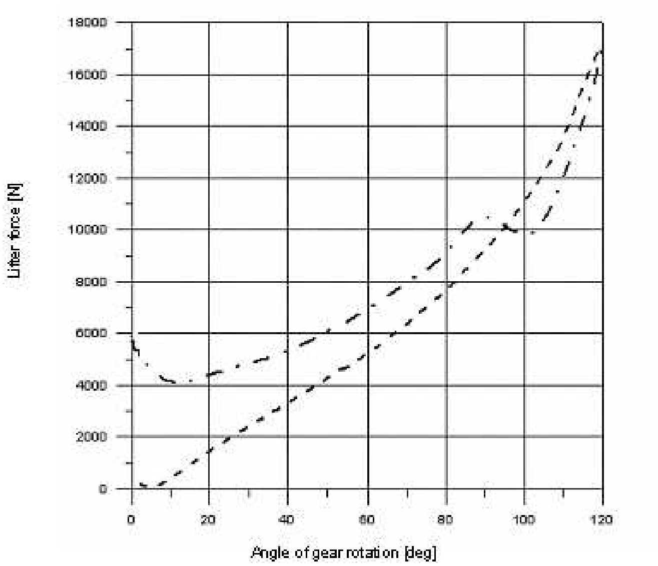

Figure 4 Force in actuator from mass (- - - -) and aerodynamic loadings (———) with angle of gear rotation

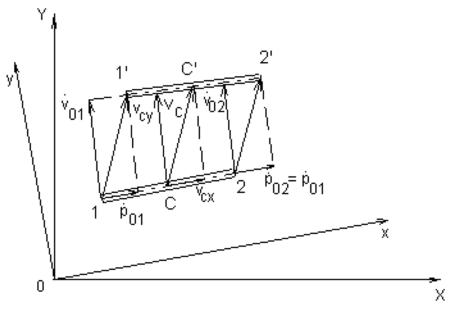

Figure 5 Displacement of a rigid finite element in global and local coordinates

The components of nodal displacement vectors 11' and 22' can be expressed in the global coordinate system XY by $\{s_0\}$ and in the local coordinate system xy by $\{\delta_0\}$

$$\{s_0\}^T = [u_{01}, w_{01}, u_{02}, w_{02}] \tag{2}$$

$$\{\delta_0\}^T = [p_{01}, v_{01}, p_{02}, v_{02}] \tag{3}$$

where elements of the above vectors are displacements of nodes 1 and 2 in X and Y or x and y directions, respectively. The vector $\{\delta_0\}$ is transformed into $\{\mathrm{s}_0\}$ by

$$\{\delta_0\} = [T]\{s_0\} \tag{4}$$

where $[T]$ is a transformation matrix

$$[T_0] = \begin{bmatrix} a & b & 0 & 0 \\ -b & a & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -b & a \end{bmatrix} \tag{5}$$

where $a = \cos\,\alpha$, $b = \sin\,\alpha$, α is the angle between local and global coordinate systems.

The shape function for the rigid body motion $[N_e]$ is taken in the form presented in [7] which ensures that the longitudinal displacement along a finite element is the arithmetic average of the displacements of its nodes and the transverse displacement of a finite element changes linearly with the length of the element:

$$[N_{e0}] = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 0 & 1-\varsigma/L & 0 & \varsigma/L \end{bmatrix} \tag{6}$$

where $0 \leq \varsigma \leq L$, L is the length of a finite element.

The correctness of the presented shape function can be checked by calculation of the kinetic energy T_e of a rigid finite element, which can be determined from the following equation:

$$T_{e0} = \frac{1}{2}\rho A \int\limits_m \dot{\mathbf{R}}_0^T \dot{\mathbf{R}}_0 d\varsigma \equiv \frac{1}{2}\rho A \int\limits_m \dot{\mathbf{r}}_0^T \dot{\mathbf{r}}_0 d\varsigma = \frac{1}{2}\left\{\dot{\delta}_0\right\}^T [M_{e0}] \left\{\dot{\delta}_0\right\} \tag{7}$$

where $\{\delta_0\}$ is given by equation (3) and assuming constant cross–sectional area A and constant material density ρ the element inertia matrix for rigid body motion $[M_e]$ is given by

$$[M_{e0}] = \rho A \int\limits_0^L [N_{e0}]^T [N_{e0}]\, d\varsigma \tag{8}$$

Taking into account that for the rigid elements the displacements in longitudinal direction are equal (i.e. $p_{01} = p_{02}$) the kinetic energy derived from equation (7) is equal to

$$T_{e0} = \frac{\rho A L}{6}\left(3\dot{p}_{01}^2 + \dot{v}_{01}^2 + \dot{v}_{02}^2 + \dot{v}_{01}\dot{v}_{02}\right) \tag{9}$$

It can be easily proved that the same result is obtained from classical mechanics of rigid bodies. For a moving stiff plane rod (Fig. 5) with the same geometric parameters as the finite element considered, the kinetic energy can be presented as a sum of energy in progressive motion with velocity of the center point C (v_C) and energy in rotational motion

$$T_{e0} = \frac{m v_C^2}{2} + \frac{I\omega^2}{2} \tag{10}$$

where moment of inertia $I = mL^2/12$ and angular velocity $\omega \equiv \dot{\alpha} = (\dot{v}_{02} - \dot{v}_{01})/L$ are measured in relation to the center C. The velocity v_C of the center point can be expressed as

$$v_C = \sqrt{\dot{p}_{01}^2 + \left(\frac{\dot{v}_{01} + \dot{v}_{01}}{2}\right)^2} \tag{11}$$

In view of the constancy of cross–sectional area and mass density of a finite element, the elements mass can be expressed as $m = \rho AL$ and both expressions on kinetic energy (i.e. eqns (9) and (10)) give the same result.

The matrix differential equation of motion for a single finite element of the mechanism is developed by using Lagrange's equations of motion. In the case of the rigid body equations of motion it must be taken into account that displacements of both nodes in the longitudinal direction of elements are equal or the difference between them results from Hook's law. In the latter case it is necessary to build the element stiffness matrix $[K_e]$ based on one–dimensional stress analysis. In order to do this the displacement vector of nodes in longitudinal direction $\{p_0\}$ is introduced (see Fig. 5). It can be transformed into element vector $\{\delta_0\}$ by a transformation matrix $[T_1]$:

$$\{p_0\} = [p_{01}, p_{02}]^T = [T_1]\{\delta_0\} \tag{12}$$

$$[T_1] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{13}$$

In the case of neglecting gravity the potential energy is restricted to strain energy. This energy is based on one–dimensional stress analysis and is as follows

$$U_{e0} = \frac{1}{2}\{p_0\}^T [k_0]\{p_0\} = \frac{1}{2}\{\delta_0\}^T [K_{e0}]\{\delta_0\} \tag{14}$$

where the element stiffness matrix $[k]$ is in the same form as for a planar truss element (Bathe, 1976):

$$[k_0] = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{15}$$

where E is the Young modulus, and $[K_e]$ is obtained from

$$[K_{e0}] = [T_1]^T [k_0][T_1] \tag{16}$$

Substituting equations on strain (14) and kinetic (7) energy into Lagrange's equations the rigid body element equation of motion are obtained in the following form

$$[M_{e0}]\left\{\ddot{\delta}_0\right\} + [K_{e0}]\{\delta_0\} = \{Q_{e0}\} \tag{17}$$

The element stiffness matrix is as follows

$$[K_e] = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

The element inertia matrix is given by

$$[M_e] = \frac{\rho AL}{12}\begin{bmatrix} 3 & 0 & 3 & 0 \\ 0 & 4 & 0 & 2 \\ 3 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 \end{bmatrix} \tag{19}$$

Taking into account that for large-displacement motion for vectors measured in different coordinate systems the following equations hold

$$\left\{\dot{\delta}_0\right\} = [T_0]\{\dot{s}_o\} \tag{20}$$

$$\left\{\ddot{\delta}_0\right\} = [T_0]\{\ddot{s}_o\} \tag{21}$$

the equations of motion for one element in global coordinates are obtained by premultiplying element equations (17) by $[T]^T$ and are as follows

$$[M_{ea}]\{\ddot{s}_0\} + [K_{ea}]\{s_0\} = \{Q_{ea}\} \tag{22}$$

where $[M_{ea}]$ and $[K_{ea}]$ are obtained from the element matrices $[M_e]$ and $[K_e]$ by:

$$[M_{ea}] = [T_0]^T [M_{e0}][T_0] \tag{23}$$

$$[K_{ea}] = [T_0]^T [K_{e0}][T_0] \tag{24}$$

Following the standard technique in expanding element equations to system size, and combining all the element equations, the global equations of motion for the system are stated as

$$[M_0]\{\ddot{x}_0\} + [K_0]\{x_0\} = \{Q_0\} \tag{25}$$

where $[M]$, $[K]$ are the system matrices obtained from $[M_{ea}]$, $[K_{ea}]$; $\{x_0\}$ is obtained from $\{s_0\}$ by taking into account the boundary conditions, and $\{Q\}$ is the external system force vector.

The presence of stiffness matrix in equations (17) and (25) is only necessary due to proper modeling of displacements of the rigid element and has practically no influence on rigid body motion (the kinetic energy of an element and consequently element inertia matrix were taken for rigid elements). If the stiffness matrix was omitted, the nodes could displace in any direction – also in longitudinal direction of the element which is impossible due to the rigidity of elements. Alternatively it is possible to introduce constraint inequalities implied on element's nodes [9].

The presented method was applied for the retraction mechanism of aircraft landing gear. The differential matrix equation (25) was solved by using Newmark method for the data presented in Tab. 1. In conducted calculations the aerodynamic forces were omitted. The time needed for the gear to retract is about 0.132 s and the retraction angle is about 2.1 rad. In order to check the correctness of the obtained results the calculations using the ADAMS system for the same beam model of landing gear have been conducted. The results are presented in Fig. 6 for the changes of the crank angle with time, and in Fig. 7 for the changes in crank angle velocity with time. From the figures it can be seen that the solutions obtained from the presented finite element method is very close to those obtained by using multibody method used in the ADAMS software.

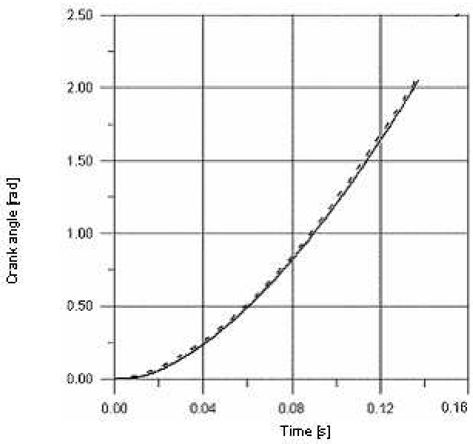

Figure 6 Crank angle versus time for FEM model (- - - -) and ADAMS model (———)

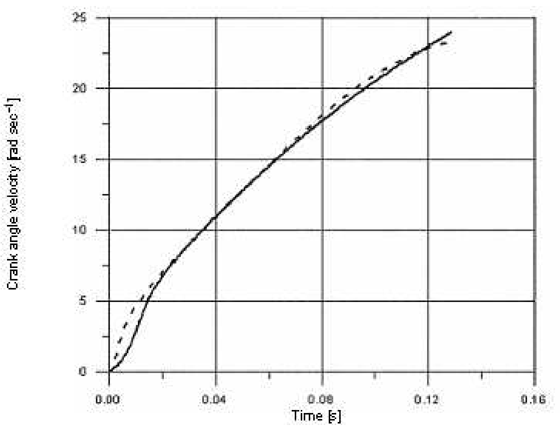

Figure 7 Crank angle velocity versus time for FEM model (- - - -) and ADAMS model (———)

The presented method is very efficient because in the case considered the dimension of coefficient matrices in eqn (25) is 4x4 – there are two moving nodes (2 and 3 in Fig. 2) and each has two d.o.f. (displacement in x and y direction). Moreover the method is not sensitive on the number of load inputs and can be used for mechanisms with loadings applied to many links.

4. Influence of friction on the gear motion

In many studies friction has often been neglected in dynamic analysis of mechanisms, however some studies [4] show that in some cases the effect of friction is quite significant.

Generally, the friction arising in the revolute joint is from two sources: due to existing normal reaction forces {f} in the bearing, and the second one due to the reaction moments at the joint. This second case cannot appear when planar mechanism is considered (i.e. all loads and internal forces act in the plane of the motion). Thus in our case the friction force will be of the first kind only and effective normal forces at the joint can be expressed as

$$F_N = \sqrt{f_x^2 + f_y^2} \tag{26}$$

where f_x and f_y are the x and y components of the reaction force $\{f\}_i$ (i is the number of the joint considered). The direction of the friction force is opposite to the direction of the relative rotation between the journal and the bearing. Thus the frictional moment M_T^i for the i-th joint can be written as:

$$M_T^i = f \times r = \mu \left| F_N \right| r \tag{27}$$

where μ is the frictional coefficient and r is the journal radius.

The model of elastic–plastic friction is assumed. Two kinds of friction can be distinguish in this model: static friction and kinetic friction. The change of friction coefficient with the slip velocity for the assumed model is presented in Fig. 8. It can be seen that for velocities between V_S and V_d the smooth passage from friction coefficient in μ_S to μ_d takes place. This is the simplified model of friction, in which friction for small speeds is approximated by linear function.

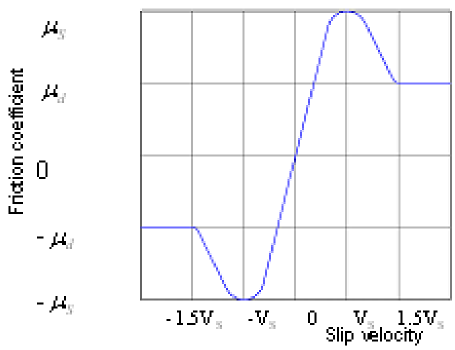

Figure 8 Change of friction coefficient with slip velocity

In all kinematic pairs the friction was modeled as follows: the coefficient of static friction was assumed to be 0.19, and the coefficient of kinetic friction was taken equal to 0.026. The pin diameter in revolute joints is equal to 30 mm, except the kinematic pair 1 (Fig. 2) in which was assumed to be 35 mm.

The results of computer simulation in ADAMS system contain the changes of crank angle of rotation, velocity and acceleration of chosen kinematic pairs of the system. They are presented in Figures 9, 10, and 11. The minimal force generated by the actuator which extorted the rotation of the gear was also calculated and is presented in Fig. 12 in function of time for the case with friction in joints and with no friction. The mass forces with overload 3g as well as aerodynamic forces are taken into account.

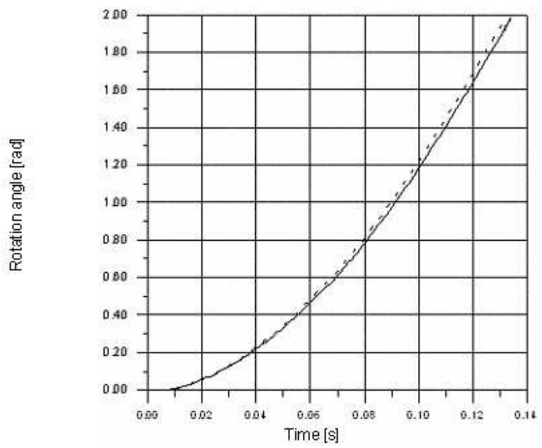

Figure 9 Rotation angle of the crank versus time with (——) and without (- - - -) friction

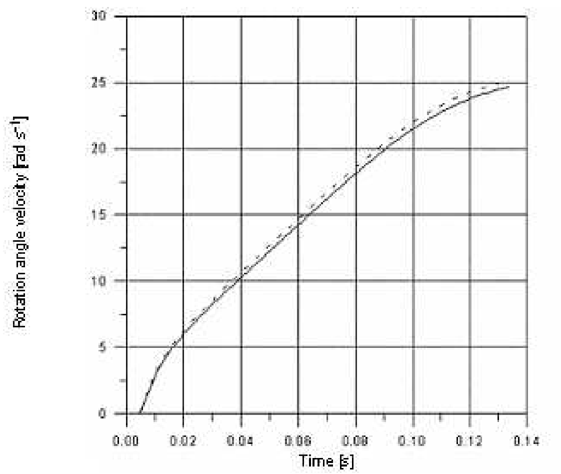

Figure 10 Rotation angle velocity of the crank versus time with (——-) and without (- - - -) friction

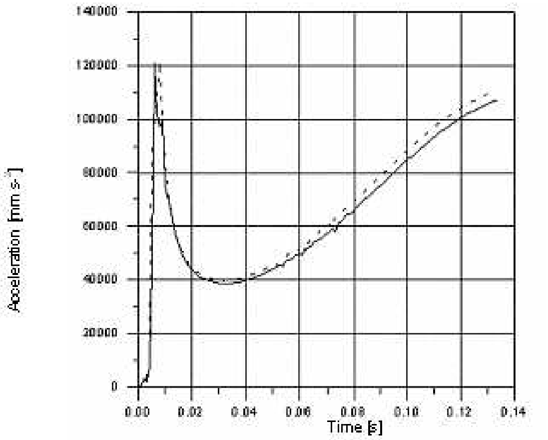

Figure 11 Acceleration of kinematic pair 2 versus time with (——-) and without (- - - -) friction

From the figures it can be seen that:

- the total time needed for retraction the gear increases about 2% when the friction is taken into account
- in order to proper model the features of the kinematic landing gear system it is necessary to introduce friction in revolute joints. The presented elastic-plastic model describes the friction phenomena with good approximation and on the other hand does not introduce discontinuities of the function describing the friction force for zero velocity. It neglects the friction forces in the case of comparative shutdown the link. This model gives reliable results for relatively higher comparative speeds, which takes place in aircraft landing gears.

5. Conclusions

Mechanism kinematics and dynamics is a very important area in the design of aircraft landing gear. Alternatively to the usage of a multibody code is the method based on finite element technique applied for rigid links. The first step taken towards the implementation of the landing gear model is the analysis of stick model motion. At this stage the rigid body model is examined and such parameters as the loads transfer to fuselage, time of retraction and extension of the gear, the minimal force needed in the actuator for retraction are considered. The analysis presented in this paper showed that the finite element model for rigid links presented in this paper is useful at this conceptual level and can be used for kinematics and dynamics of mechanisms such as retraction gears i.e. with loads applied to many links and taking into account G-forces. Alternatively this process can be conducted by derivation of equations of motion with the use of equations of classical mechanics. However in that case the equations of motion should be derived independently for each

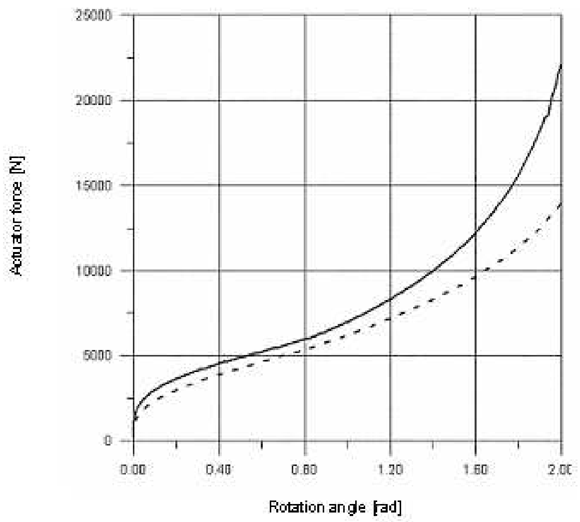

Figure 12 Minimal actuator force needed for gear rotation with (—–) and without (- - - -) friction forces versus rotation angle

mechanism and in the case of loads applied to more than one link the loadings must be transferred to general coordinates.

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Appendix

The equation of motion for four bar linkage - the drive torque is applied to the crank and the follower. Some notations used in this Appendix are shown in Fig. A1.

Fig. A1. Four bar linkage

For rigid links the system has one degree of freedom. As independent coordinate the crank angle Θ_2 is taken (Fig. A1). Equations of motion are as follows

$$M(\Theta_2)\varepsilon = Q(\Theta_2) + C(\Theta_2)\varpi^2$$

where:

ε – the angular acceleration (second derivative of generalized coordinate – the angle of crank $\ddot{\Theta}_2$),

ϖ – the angular speed (first derivative of generalized coordinate $\dot{\Theta}_2$),

M – the words standing near ε, C - the expressions standing near ϖ^2, Q - the free words. The coefficients M, Q, and C are the function of position of mechanism (the angle of crank Θ_2):

$$\begin{aligned}
M(\Theta_2) &= \frac{A_2}{A_3}l_2 + \frac{A_4}{A_3}l_4\kappa_4^2 + l_3\left[3 + \kappa_3^2 + 3\kappa_3\cos(\Theta_2 - \Theta_3)\right] \\
&\quad + \frac{l_5^2(m_5 + 3m_{5k})}{l_2^2\rho A_3} \\
Q(\Theta_2) &= \frac{3T_c}{\rho l_2^2 A_3} - \frac{A_4 l_4^2}{A_3 l_2}\kappa_4\alpha_4 - \frac{l_3^2}{l_2}\kappa_3\alpha_3 \\
&\quad - 1.5\frac{l_3^2}{l_2}\left[\alpha_3\cos(\Theta_2 - \Theta_3) + \Omega_3^2\sin(\Theta_2 - \Theta_3)\right] \\
C(\Theta_2) &= 1.5 l_3\kappa_3\sin(\Theta_2 - \Theta_3)
\end{aligned}$$

where:

A_2, A_3, A_4 – cross–sectional area of respectively: crank, coupler, follower,

ρ – material density,

m_5 – the mass of shock absorber link, m_{5k} - the mass of wheel,

l_2, l_3, l_4 – the length of successive links,

l_5 – the link of shock absorber with wheel.

The dependence between angles of four–bar linkage:

the diagonal of four–bar linkage:

$$
\begin{aligned}
a &= \sqrt{l_2^2 + l_1^2 - 2l_1 l_2 \cos(\Theta_2)} \\
\alpha &= \arccos \frac{l_4^2 + a^2 - l_3^2}{2al_4} \\
\beta &= \arctan \frac{l_2 \sin(\Theta_2)}{l_1 - l_2 \cos(\Theta_2)} \\
\gamma &= \arccos \frac{l_3^2 + a^2 - l_4^2}{2al_3} \\
\Theta_3 &= \gamma - \beta \\
\Theta_4 &= -(\alpha + \beta) \\
\varepsilon_1 &= \ddot{\Theta}_1 \\
\Omega_2 &= \dot{\Theta}_2 \\
\Omega_3 &= \dot{\Theta}_3 = \frac{l_2}{l_4} \kappa_3 \Omega_2 \\
\kappa_3 &= \frac{\sin(\Theta_2 - \Theta_4)}{\sin(\Theta_4 - \Theta_3)} \\
\Omega_4 &= \dot{\Theta}_4 = -\frac{l_2}{l4} \kappa_4 \Omega_2 \\
\kappa_4 &= \frac{\sin(\Theta_2 - \Theta_3)}{\sin(\Theta_4 - \Theta_3)} \\
\alpha_3 &= \frac{l_3 \Omega_3^2 \cos(\Theta_3 - \Theta_4) + l_4 \Omega_4^2 + l_2^{\Omega}{}_2^2 \cos(\Theta_2 - \Theta_4)}{l_3 \sin(\Theta_4 - \Theta_3)} \\
\alpha_4 &= \frac{l_3 \Omega_3^2 + l_4 \Omega_4^2 \cos(\Theta_4 - \Theta_3) + l_2 \Omega_1^2 \cos(\Theta_2 - \Theta_3)}{l_4 \sin(\Theta_3 - \Theta_4)}
\end{aligned}
$$

The generalized moment without taking into account gravity:

$$T_C = T_1 + T_2 \frac{\partial \theta_4}{\partial \theta_2}$$

where:

T_1, T_2 – external torques applied to crank and follower

and

$$\frac{\partial \theta_4}{\partial \theta_2} = -\left(\frac{\partial \alpha}{\partial \theta_2} + \frac{\partial \beta}{\partial \theta_2}\right) = \frac{\sin \Theta_2 l_1 l_2 (a^2 + l_3^2 - l_4^2)}{2a^{3/2} l_4 \sqrt{1 - \frac{(a^2 - l_3^2 + l_4^2)^2}{4a^2 l_4^2}}} + \frac{l_1 l_2 \cos \Theta_2 - l_2^2}{a^2}$$

Generalized torque with regard to the gravity moment – it is assumed that between global coordinate system and the vector of gravy is angle χ , where $\chi = \pi/2 - \delta$ (see Fig. 3):

$$T_C = T_1 + T_2 \frac{\partial \theta_4}{\partial \theta_2} - T_{2g} - T_{3g} - T_{4g} + T_{5g}$$

where:

T_{2g}, T_{3g}, T_{4g} , T_{5g} – torques coming from gravity of successive links as well as from wheel with shock absorber:

$$\begin{aligned} T_{2g} &= 0.5m_2l_2g\cos(\Theta_2+\chi) \\ T_{3g} &= m_3g\left(l_2\cos\chi + 0.5l_3\cos(\Theta_3+\chi)\frac{\partial\theta_3}{\partial\theta_2}\right) \\ T_{4g} &= 0.5m_4gl_4\cos(\Theta_4+\chi)\frac{\partial\theta_4}{\partial\theta_2} \\ T_{5g} &= 0.5(m_5+m_{5k})gl_5\cos(\Theta_2+\psi+\chi) \end{aligned}$$

$$\frac{\partial\theta_3}{\partial\theta_2} = \frac{\partial\gamma}{\partial\theta_2} - \frac{\partial\beta}{\partial\theta_2} = -\frac{\sin\Theta_2 l_1 l_2(a^2-l_3^2+l_4^2)}{2a^{3/2}l_3\sqrt{1-\frac{(a^2+l_3^2-l_4^2)^2}{4a^2l_3^2}}} - \frac{l_1l_2\cos\Theta_2 - l_2^2}{a^2}$$

m_2, m_3, m_4 – masses of individual links,
m_5 - the mass of the link with shock absorber,
m_{5k} - the mass of wheel